RESUME OF MATHEMATICS

1. **STANDARD OF THE PAPER**

The Chief Examiners for Mathematics (Core) and Mathematics (Elective) reported that the standard of the papers compared favourably with that of the previous year.

2. **CANDIDATES' PERFORMANCE**

The Chief Examiners for both Mathematics (Core) and Mathematics (Elective) stated that the performance of candidates was better than the previous year.

3. **CANDIDATES' STRENGTHS**

The Chief Examiner for Mathematics (Core) listed some of the strengths of candidates as ability to:

(i) express decimals in their standard forms;
(ii) complete tables for trigonometric equations and draw the associated graph;
(iii) solve questions involving transformation;
(iv) convert numbers from one base to the other;
(v) express irrational numbers in their surd form.

The Chief Examiner for Mathematics (Elective) also listed the strengths of the candidates as ability to:

(i) rationalise a given surd;
(ii) find the mean and standard deviation from a given data;
(iii) find the resultant and magnitude of given vectors;
(iv) resolve given forces into component form;
(v) find the determinant and inverse of a given matrix.

4. **CANDIDATES' WEAKNESSES**

The Chief Examiner for Mathematics (Core) listed some of the weaknesses of candidates as difficulty in:

(i) translating word problems into mathematical statements;
(ii) solving probability related problems;
(iii) solving basic computation without the use of calculator;
(iv) solving problems involving mensuration;
(v) showing evidence of reading from a graph.

The Chief Examiner for Mathematics (Elective) listed some of the weaknesses of candidates as difficulty in:

(i) finding the coordinates of a point using internal division of a line in a given ratio;
(ii) solving problems on mechanics;
(iii) finding the range of a given inequality;

(iv) using trigonometric identities to solve trigonometry related problems.

5. SUGGESTED REMEDIES

The Chief Examiners for both Mathematics (Core) and Mathematics (Elective) suggested that teachers should:

(i) give equal attention to the whole syllabus rather than specializing in few areas in the syllabus;

(ii) expose candidates to a lot of questions as exercises for them to have a good command on the topics in the syllabus;

(iii) coach students on the importance of accuracy so that they would work with at least four decimal places and round off answers after the whole calculation;

(iv) stress on the need to read and understand the demand of the question before answering.

MATHEMATICS CORE (2)

1. **GENERAL COMMENTS**

The Chief Examiner stated that the standard of the questions compared favourably with that of previous years and that the performance of the candidates was better than that of the previous year.

2. **CANDIDATES' STRENGTHS**

The Chief Examiner for Mathematics (Core) listed some of the strengths of candidates as ability to:

(i) express decimals in their standard forms;
(ii) complete tables for trigonometric equations and draw the associated graph;
(iii) solve questions involving transformation;
(iv) convert numbers from one base to the other.

3. **CANDIDATES' WEAKNESSES**

The Chief Examiner for Mathematics (Core) listed some of the weaknesses of candidates as difficulty in:

(i) translating word problems into mathematical statements;
(ii) solving probability related problems;
(iii) solving basic computation without the use of calculator;
(iv) solving problems involving mensuration;
(v) showing evidence of reading from a graph.

4. **SUGGESTED REMEDIES**

(i) Teachers should stress on the need for candidates to read and understand the demands of questions they attempt.
(ii) Sufficient exercises should be given on word-problems for students to practice.
(iii) The concept of mensuration must be taught well in schools.
(iv) Students should be encouraged to show evidence of reading from graphs.
(v) Teachers should teach students the basic concepts in probability.

5. **DETAILED COMMENTS**

Question 1

(a) Without using mathematical tables or calculators, evaluate $\frac{0.015\times0.567}{0.05\times0.189}$, leaving the answer in standard form.

(b) If $\frac{5y-x}{8y+3x}=\frac{1}{5}$, find, correct to two decimal places, the value of $\frac{x}{y}$.

(a) Candidates were asked not to use calculator or tables but after expressing the given decimals in standard forms, most candidates used the calculator or tables to simplify their results. Candidates were required to solve part (a) as:

$= \frac{0.015 \times 0.567}{0.05 \times 0.189}$

$= \frac{15 \times 10^{-3} \times 567 \times 10^{-3}}{5 \times 10^{-2} \times 189 \times 10^{-3}}$

$= \frac{3 \times 3 \times 10^{-6}}{1 \times 10^{-5}}$

$= 9 \times 10^{-6+5}$

$= 9 \times 10^{-1}$

(b) Candidates were able to simplify the expression but could not write the required ratio, and correct to two decimal places. Candidates were expected to solve part (b) as:

$\frac{5y-x}{8y+3x} = \frac{1}{5}$

$5(5y - x) = 8y + 3x$

$25y - 5x = 8y + 3x$

$25y - 8y = 5x + 3x$

$17y = 8x$

$\frac{17}{8} = \frac{x}{y}$

$\frac{x}{y} = 2.125$

$\frac{x}{y} = 2.13$ (2d.p.)

Question 2

(a) ***Z* varies directly as *x* and inversely as twice the cube root of *y*. If *Z* = 8, when *x* = 4 and *y* = $\frac{1}{8}$, find the relation for *y* in terms of *x* and *z*.**

(b) **Factorize completely: $4b^2 - ab + (a + 9b)^2 - a^2$.**

Candidates were able to find the constant of variation (*k*) but could not simplify the resulting equation after substituting *k* into the equation.

Candidates were required to solve part (a) as follows:

$z\alpha \ \frac{x}{2(\sqrt[3]{y})}$

$z = \frac{kx}{2(\sqrt[3]{y})}$

$8 = \frac{4k}{2\left(\sqrt[3]{\frac{1}{8}}\right)}$

$8 = \frac{4k}{2\left(\frac{1}{2}\right)}$

$4k = 8 \quad \Rightarrow k = 2$

$\mathfrak{z} = \frac{2x}{2(\sqrt[3]{y})}$

$y = \frac{x^3}{\mathfrak{z}^3}$

$y = \left(\frac{x}{\mathfrak{z}}\right)^3$

Candidates could not expand $(a+9b)^2$ and hence could not factorize the given expression completely.

Candidates were required to solve part (b) as follows:

$4b^2 - ab + (a + 9b)^2 - a^2$

$= 4b^2 - ab + a^2 + 18ab + 81b^2 - a^2$

$= 4b^2 + 81b^2 - ab + 18ab$

$= 85b^2 + 17ab$

$= 17b\,(5b + a)$

<u>Question 3</u>

(a) Solve $\frac{5x-7}{6} + \frac{2x-3}{4} = -\frac{2}{3}$.

(b) Evaluate: $\frac{\sqrt{28}+\sqrt{343}}{2\sqrt{63}} + \frac{5}{3}$.

(a) Candidates were able to solve for the missing variable x.

Candidates were required to solve part (a) as follows;

$\frac{5x-7}{6} + \frac{2x-3}{4} = \frac{-2}{3}$

$4\,(5x - 7) + 6\,(2x - 3) = 8(-2)$

$20x - 28 + 12x - 18 = -16$

$32x = -16 + 46$

$32x = 30$

$x = \frac{30}{32}$

$= \frac{15}{16}$

(b) Candidates could write the given irrational numbers in their surd forms but could not evaluate the resulting expression.

Candidates were required to solve part (b) as follows:

$\frac{\sqrt{28}+\sqrt{343}}{2\sqrt{63}} + \frac{5}{3}$

$= \frac{2\sqrt{7}+7\sqrt{7}}{6\sqrt{7}} + \frac{5}{3}$

$= \frac{9\sqrt{7}}{6\sqrt{7}} + \frac{5}{3}$

$= \frac{9}{6} + \frac{5}{3}$

$= \frac{9+10}{6}$

$= \frac{19}{6}$

$= 3\frac{1}{6}$

<u>Question 4</u>

A car dealer made a profit of 22.5% by selling a car for GH¢58,000.00. Find, correct to two decimal places, the percentage profit if the car had been sold for GH¢61,200.00.

Candidates were able to find the given percentage, but a few could not write their percentage to two decimal places as required.

Candidates were required to solve question 4 as follows:

Let x = C.P, S.P = GH¢58,000.0

Profit % = 22.5%

$\frac{58{,}000-x}{x} \times 100 = 22.5$

5,800,000 – 100x = 22.5x

5,800,000 = 122.5x

$x = \frac{5{,}800{,}000}{122.5}$

= GH¢ 47, 346.94

Profit % = $\frac{61{,}200-47{,}346.94}{47{,}346.94} \times 100$

$= \frac{13{,}853.06}{47{,}346.94} \times 100$

= 29.2586

= 29.26% (2 d.p.)

<u>Question 5</u>

(a) A number is chosen at random from Q = {1, 2, 3, …., 10}. Find the probability that the chosen number is either a prime factor of 42 or a multiple of 3.

(b) If $110_x = 1020_{four}$, find the value of x.

(a) Candidates were able to write prime factors of 42 and multiples of 3 from the given set but could not write the required probabilities. Most candidates did not realize the two events were independent events and hence could not find the required probability.

Candidates were required to solve part (a) as follows:
Q = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
Prime factors of 42 = {2, 3, 7}
Multiples of 3 = {3, 6, 9}
P (Prime factors of 42 or multiples of 3)
$= \frac{3}{10} + \frac{3}{10} - \frac{1}{10}$
$= \frac{1}{2}$

(b) Candidates were able to convert both bases to base ten and found the value of x. A few, however, could not interpret the result they got after finding the two values of x.

Candidates were required to solve part (b) as follows:
$110_x = 1020_{\text{four}}$
$(1 \times x^2) + (1 \times x^1) + (0 \times x^\circ) = (1 \times 4^3) + (0 \times 4^2) + (2 \times 4^1) + (0 \times 4^\circ)$
$x^2 + x + 0 = 64 + 0 + 8 + 0$
$x^2 + x - 72 = 0$
$x^2 - 8x + 9x - 72 = 0$
$x(x - 8) + 9(x - 8) = 0$
$(x - 8)(x + 9) = 0$
$x = -9$ or 8
$x = 8$ (since x cannot be negative).

Question 6

(a) If $\boldsymbol{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\boldsymbol{b} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ and $\boldsymbol{r} = \boldsymbol{a} + \frac{1}{2}(\mathbf{a} - \boldsymbol{b})$, find:

***(i)* $\boldsymbol{r}$;**

(ii) $|\boldsymbol{r}|$.

(b) Given that $a = bc$ and $n = \frac{mk}{ec}$,

(i) express k in terms of a, b, e, m and n;

(ii) find, correct to three significant figures, the value of k, when $a = \frac{1}{2}$, $b = -4$, $e = 3$, $m = 7$ and $n = -5$

(a) Candidates were able to substitute the given vectors into the vector equation, some however could not multiply by the scalar $\frac{1}{2}$.

Candidates were required to solve part (a) as follows:

(i) $r = \begin{pmatrix}2\\3\end{pmatrix} + \frac{1}{2}\left[\begin{pmatrix}2\\3\end{pmatrix} - \begin{pmatrix}4\\5\end{pmatrix}\right]$

$= \begin{pmatrix}2\\3\end{pmatrix} + \frac{1}{2}\begin{pmatrix}-2\\-2\end{pmatrix}$

$= \begin{pmatrix}2\\3\end{pmatrix} + \begin{pmatrix}-1\\-1\end{pmatrix}$

$= \begin{pmatrix}1\\2\end{pmatrix}$

(ii) $|r| = \sqrt{1^2 + 2^2}$

$= \sqrt{5}$ units

(b) Candidates were able to substitute c into the given equation and made k the subject. Candidates were also able to substitute the given values into their resulting equations but were not able to write their final answers to the required significant figures.

Candidates were required to solve part (b) as follows:

(i) $a = bc$, $n = \frac{mk}{ec}$, $c = \frac{a}{b}$

$n = \frac{mk}{\frac{ae}{b}}$

$n = \frac{bmk}{ae}$

$k = \frac{aen}{bm}$

(ii) $k = \frac{\frac{1}{2}\times 3\times(-5)}{-4\times 7}$

$= \frac{-\frac{15}{2}}{-28}$

$k = \frac{15}{56}$

$= 0.268$ (3 s. f.)

<u>Question 7</u>

(a) Copy and complete the table of values for $y = 5\sin x + 9\cos x$ for $0° \leq x \leq 150°$.

x	0°	30°	60°	90°	120°	150°
y		**10.3**			**-0.2**	

(b) Using a scale of 2cm to 30° on the x – axis and 2cm to 2 units on the y – axis, draw the graph of $y = 5\sin x + 9\cos x$ for $0° \leq x \leq 150°$.

(c) Use the graph to solve the equations:

(i) $5\sin x + 9\cos x = 0$;

(ii) $5\sin x + 9\cos x = 2$;

(d) Using the graph, find, the value of y when $x = 45^o$.

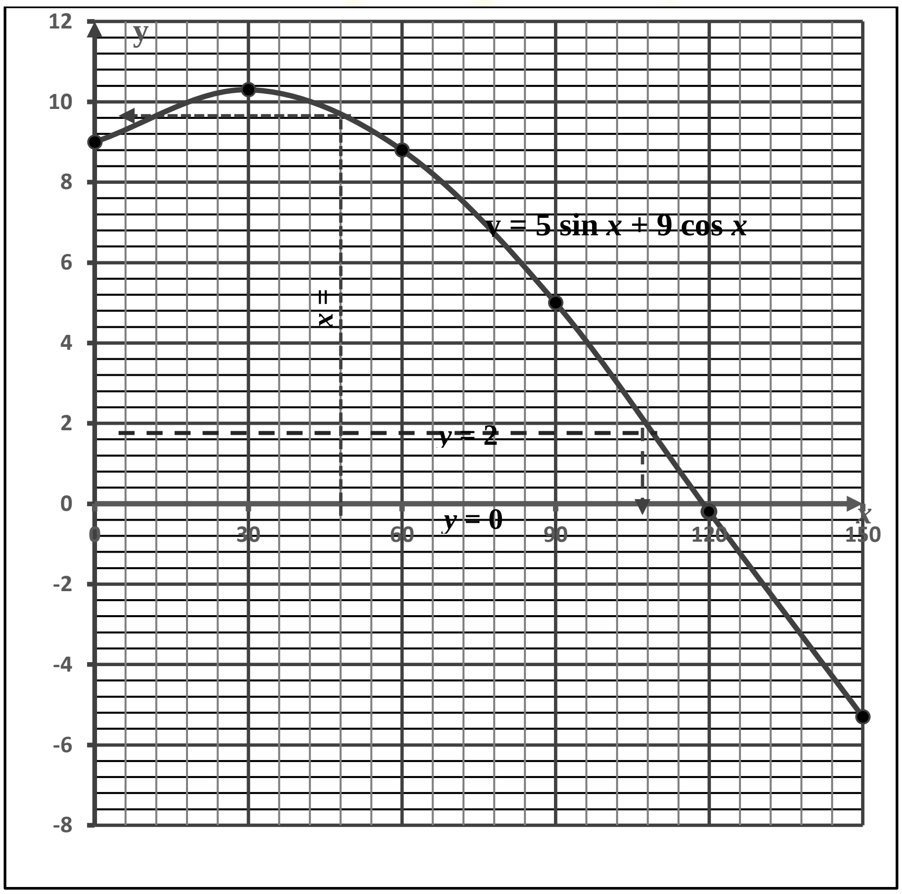
Candidates were able to complete the table of values for the given trigonometric equation, but a few could not plot their values accurately. Some also were not able to show evidence of their readings on the graph.

Candidates were expected to solve question 7 as follows:

(a)

x	0^o	30^o	60^o	90^o	120^o	150^o
y	**9.0**	10.3	**8.8**	**5.0**	-0.2	**-5.3**

(b)



(c) (i) $y = 0$, $\therefore x = 118.5^o \pm 3$

(ii) $y = 2$ $\therefore x = 108^o \pm 3$

(d) From the graph when $x = 45^o$, $y = 9.8 \pm 0.2$

<u>Question 8</u>

(a) Using ruler and a pair of compasses only, construct:

(i) the quadrilateral *ABCD* such that $|AB|$ = 6.5cm,$|BC|$= 9cm,$|AD|$= 4cm, $\angle ABC$ = 60°and$\angle BAD$ = 120°;

(ii) the perpendicular bisectors of $\overline{BC}$ and $\overline{CD}$.

(b) Locate the point of intersection, *T*, of the two bisectors in 8 (a)(ii).

(c) With the point *T* in 8(*b*) as centre, draw a circle to pass through the vertices *B*,*C*, and *D*.

(d) Measure:

(i) $|BT|$;

(ii) $|CD|$.

Some candidates did not indicate the arcs for the various measurements taken. Candidates were able to construct 120^{o} and 60^{o} but a few did not show or indicate the arcs for the measured lengths. A few could not also bisect the two lines required to get the point *T* and therefore could not draw the required circle passing through the points *B*, *C* and *D*.

Candidates were required to work question 8 as follows:

(a) Construct line segment, $|AB|$= 6.5 cm

Construct $\angle ABC$ = 60^{o} and $\angle BAD$ = 120^{o}.

$|BC|$= 9cm and$|AD|$ = 4cm

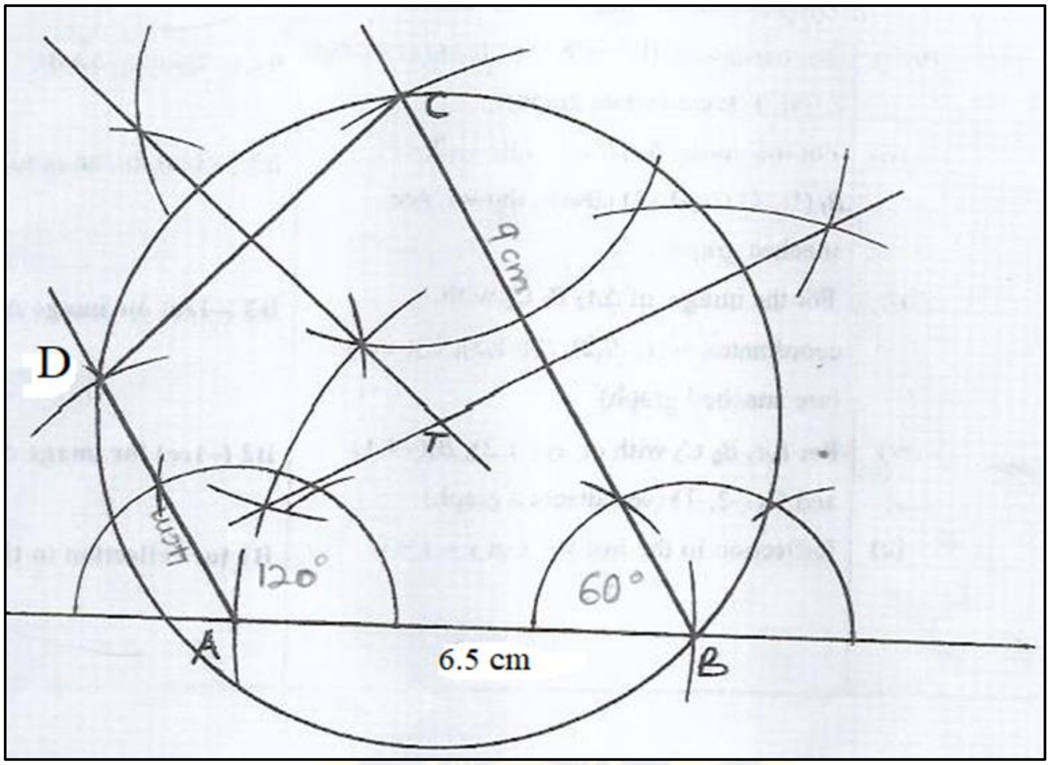
Bisect $\overline{BC}$and$\overline{CD}$

(b) Locate *T* at the point of intersection of the two bisectors.

(c) Construct a circle passing through vertices *B, C and D.*

(d) (i) $|BT|$ = 6cm ± 0.1cm

(ii) $|CD|$ = 4.8cm ± 0.1cm



Not drawn to scale

<u>Question 9</u>

(a) **Using a scale of 2cm to 1 unit on both axes, draw on a sheet of graph paper, two perpendicular axes $0x$ and $0y$ for $-5 \leq x \leq 5$ and $-5 \leq y \leq 5$.**

(b) **Draw on the same graph sheet, indicating clearly all vertices and their coordinates:**

(i) ΔABC with vertices A (2, 1), B (1, 4) and C (–1, 2);

(ii) the image $\Delta A_1B_1C_1$ of ΔABC under a reflection in the line $y = 0$, where A$\rightarrow A_1, B \rightarrow B_1$ and C$\rightarrow C_1$,

(iii) the image $\Delta A_2B_2C_2$ of ΔABC under a translation by the vector $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$, where A$\rightarrow A_2$, $B \rightarrow B_2$ and $C \rightarrow C_2$.

(iv) the image $\Delta A_3B_3C_3$ of ΔABC under an anticlockwise rotation of 90° about the origin, where $A\rightarrow A_3, B \rightarrow B_3$ and $C \rightarrow C_3$.

(c) **What single transformation maps $\Delta A_1B_1C_1$ onto $\Delta A_3B_3C_3$, where $A_1 \rightarrow A_3, B_1 \rightarrow B_3$ and $C_1 \rightarrow C_3$?**

Most of the candidates were able to draw the given triangle *ABC* and reflect *ABC* to get triangle $A_1B_1C_1$. Candidates were able to translate triangle *ABC* to triangle $A_2B_2C_2$ and also rotate triangle *ABC* to get triangle $A_3B_3C_3$.

A few candidates were able to write the single transformation mapping triangles $A_1B_1C_1$ to $A_3B_3C_3$. Candidates were required to work question 8 as follows:

(a) Draw the two perpendicular axes *Ox* and *Oy* (see attached graph)

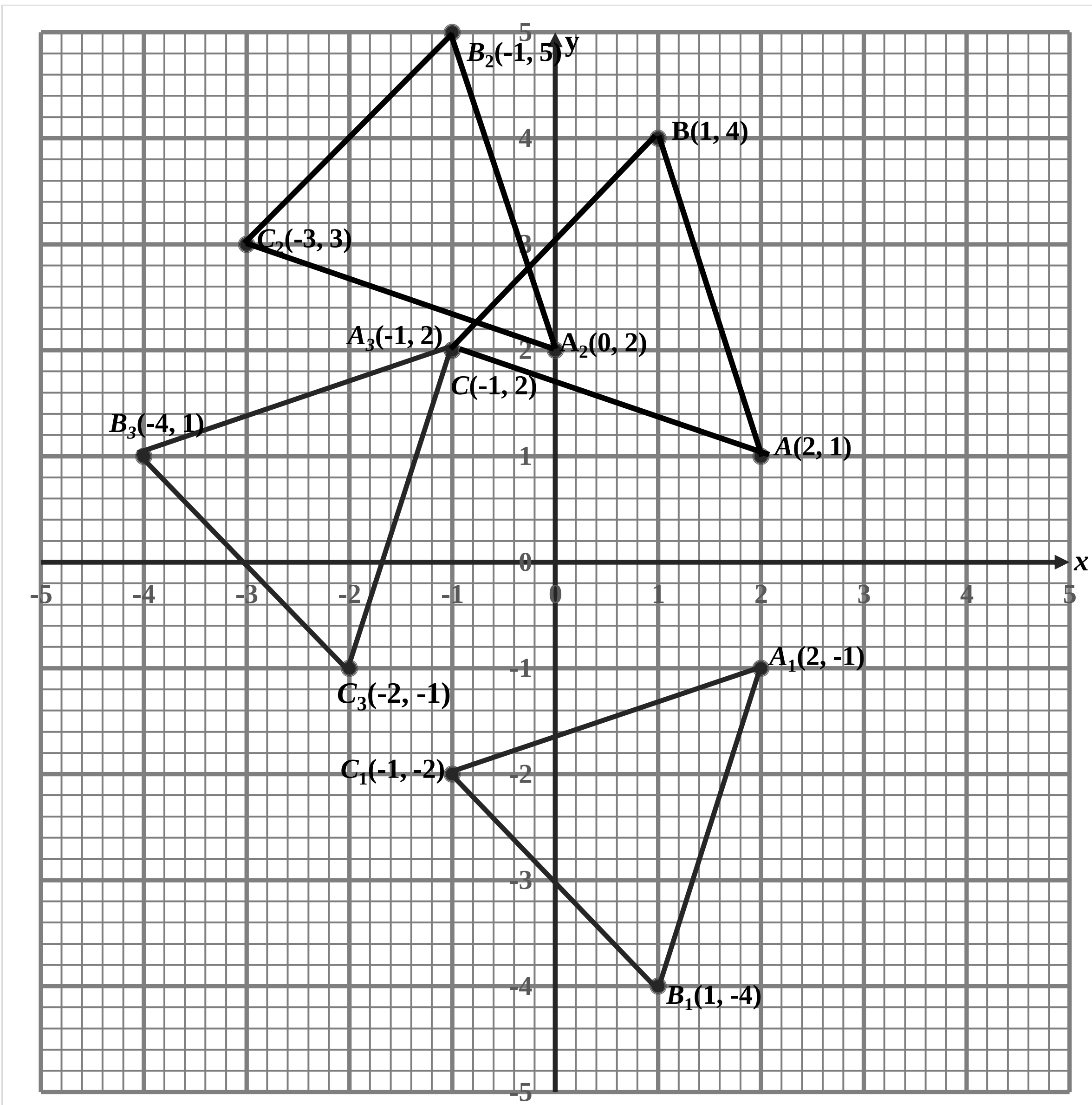
(b) (i) For triangle ΔABC with A (2,1), *B* (1,4) and *C* (-1,2) (see attached graph)

(ii) For the image $\Delta A_1B_1C_1$ with A_1(2, -1), B_1 (1, - 4), C_1(-1, -2) clearly shown (see attached graph)

(iii) For the image of $\Delta A_2B_2C_2$ with coordinates of A_2 (0,2), B_2 (-1,5), C_2 (-3,3) (see attached graph)

(iv) For $\Delta A_3\, B_3C_3$ with of A_3 (-1,2), B_3 (-4,1) and C_3 (-2, -1) (see attached graph)

(c) Reflection in the line $y = x$ or $y - x = 0$.



<u>Question 10</u>

(a) In a class of 50 students, 24 like football, 21 basketball and 18 crickets. Six like football and basketball only, 3 like basketball only, 5 like all the three games and 14 did not like any of the three games.

(i) Illustrate this information on a *Venn diagram*.

(ii) Find the number of students who like:

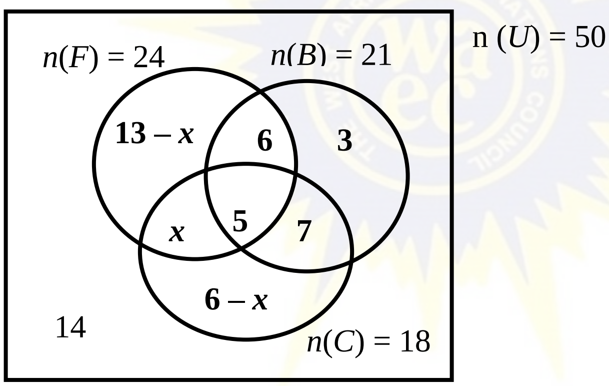
(α) football and cricket only;

(β) exactly one of the games.

(b) If (3 – *a*), 6, (7 – 5*a*) are consecutive terms of a Geometric Progression (G.P.) with common ratio *r* > 0, find the values of *a*.

(a) Candidates were able to draw the Venn diagram representing the given information but could not label the given sets correctly, e.g. instead of $n(U) = 50$, they would write U=50 etc.

Generally, candidates were able to find the given sets required.
The candidates were required to solve part (a) as follows:



(ii) (α) Let x = students who like football and cricket only

$13 - x + 6 + 5 + x + 3 + 7 + 6 - x + 14 = 50$

$x = 54 - 50$

$x = 4$

(β) Exactly one of the games = 9 + 3+ 2 = 14

(b) Candidates could write both equations comparing common ratio and solved to find the variable *a*. A few considered only the value of *a* that yielded a positive common ratio.

The candidates were required to solve part (b) as follows:

$\frac{6}{3-a} = \frac{7-5a}{6}$

$(3 - a)(7 - 5a) = 6 \times 6$

$5a^2 - 22a - 15 = 0$

$(a - 5)(5a + 3) = 0$

$a = 5$ or $a = -\frac{3}{5}$.

Question 11

(a) **Two passenger trains, *A* and *B*, 450 km apart, start to move towards each other at the same time and meet after 2 hours. If train B, travels $\frac{8}{7}$ as fast as train *A*. Find the speed of each train.**

(b) **A solid cube of side 8cm was melted to form a solid circular cone. The base radius of the cone is 4cm. Calculate, correct to one decimal place, the height of the cone [Take $\pi = \frac{22}{7}$].**

(a) The few candidates who attempted this question were able to find the speeds of the two trains *A* and *B*.

The candidates were required to solve part (a) as;

Let speed of train A = x

Speed of train B = $\frac{8}{7} \times x$

Distance = speed × time

Distance by A $\Rightarrow x \times 2 = 2x$

Distance by B $\Rightarrow \frac{8x}{7} \times 2 = \frac{16x}{7}$

Distance covered by $A + B$ = 450km

$2x + \frac{16x}{7} = 450$

$14x + 16x = 3150$

$30x = 3150$

$x = 105$

Speed of train A = 105kmh^{-1}

Speed of train $B = \frac{8}{7} \times 105$ kmh^{-1}

= 120kmh^{-1}

(b) Candidates were able to find the volume of the cube but could not find the volume of the cone. They were expected to equate the volume of the cube to the volume of the cone and use the ensuing equation to find the *height* of the cone.

The candidates were required to solve part (b) as follows;

Volume of cube = 8^3

= 512cm^3

Volume of cone = $\frac{1}{3} \times \pi r^2 h$

= $\frac{1}{3} \times \frac{22}{7} \times 4^2 \times h$

= $16.762h$

Volume of cube = volume of cone

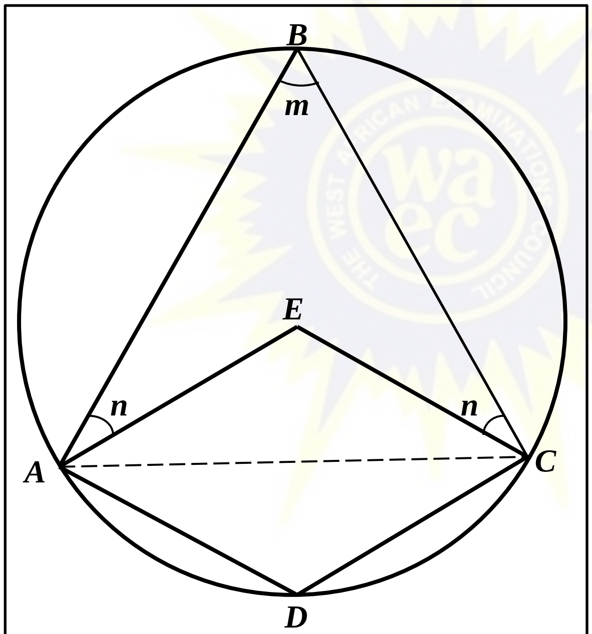
$512 = 16.762h$

$h = \frac{512}{16.762}$

$h = 30.545$

Height of cone = 30.5cm (l d.p.)

<u>Question 12</u>



NOT DRAWN TO SCALE

(a) The diagram shows a circle *ABCD* with centre *E*. Quadrilateral *EADC* is a rhombus, $\angle BAE = \angle ECB = n$ and $\angle ABC = m$. Find:

(i) *m*;

(ii) *n*.

(b) Find the quadratic equation whose roots are $\frac{3}{4}$ and -4.

(a) Candidates were able to find the missing angles. A few candidates were also using their own variables instead of m and n stated in the question and were not able to conclude by linking their chosen variables with those stated in the question.

The candidates were required to solve part (a) as follows;

$\angle AEC = 2m$

$\angle ADC = 180° – \text{m}$

$2m = 180° – m$

$3m = 180°$

$m = 60°$

$\angle AEC = 2m = 2 \times 60^o = 120^o$

ΔEAC is isosceles with base angles EAC and ECA

$= ½ (180° – 120°) = 30^o$

$\angle EAC = \angle ECA = 30^o$

From ΔABC

$m + \angle BAD + \angle BCD = 180^o$

$60° + (n + 30°) + (n + 30°) = 180^o$

$2n + 120° = 180^o$

$2n = 60^o$

$n = 30^o$

(b) Candidates were able to write the quadratic equation using the sum of roots and the product of roots. A few, however, in the process of simplifying had problems with the manipulation of the operational signs.

The candidates were required to solve part (b) as follows;

Sum of roots = $\frac{3}{4} + (– 4) = -\frac{13}{4}$

Product of roots = $\frac{3}{4} \times (- 4) = –3$

$x^2 + \frac{13}{4}x – 3 = 0$

$4x^2 + 13x – 12 = 0$

<u>Question 13</u>

(a) The fourth term of an Arithmetic Progression (A.P.) is one less than twice the second term. If the sixth term is 7, find the first term.

(b) A clerk spends $\frac{1}{5}$,$\frac{1}{3}$ and $\frac{1}{8}$ of his annual salary on rent, transport, and entertainment respectively. If after all these expenses he had GH¢4,100.00 left, find how much he earns per annum.

(c) Given that $f{:}x \rightarrow 2x^2 – 8x + 5$ and $g{:}\ x \rightarrow x – 2$; Find:

(i) $f(-3)$;

(ii) **the value of** x **such that** $f(x) = g(x)$.

(a) Candidates were able to find the first term of the Arithmetic Progression but a few of them could not substitute to find the common difference that was required.

The candidates were required to solve part (*a*) as follows;

$U_4 = 2U_2 - 1$

$a + 3d = 2(a + d) - 1$

$-a + d = -1$ ……... (1)

$U_6 = 7$

$a + 5d = 7$ ………… (2)

(1) + (2): $6d = 6$

$d = 1$ and $a = 2$

(b) Candidates were able to find the fraction left but in the process of finding the annual salary, a few of them were writing the given amount in hundreds of thousands and some also wrote annual salary using the old cedis symbol (¢) omitting the 'GH' and the decimal as well.

The candidates were required to solve part (b) as follows;

Total fraction = $\frac{1}{5} + \frac{1}{3} + \frac{1}{8} = \frac{79}{120}$

Fraction of salary left = $1 - \frac{79}{120}$

$= \frac{41}{120}$

Let x = annual salary

$\frac{41}{120} \times x = 4{,}100$

$x = \frac{120}{41} \times 4{,}100$

Annual salary = GH¢12,000.00

(c) Candidates were able to substitute to find $f(-3)$ and also found the values of x for which the two functions are the same.

The candidates were required to solve part (c) as follows;

(i) $f(-3) = 2(-3)^2 - 8(-3) + 5$

$= 18 + 24 + 5$

$= 47$

(ii) $f(x) = g(x)$

$2x^2 - 8x + 5 = x - 2$

$2x^2 - 9x + 7 = 0$

$(x - 1)(2x - 7) = 0$

$x = 1$ or $x = 3.5$

FURTHER MATHEMATICS/MATHEMATICS (ELECTIVE) 2

1. **GENERAL COMMENTS**

The Chief Examiner for Mathematics (Elective) reported that the standard of the paper compared favourably with that of the previous year and that the performance of candidates was better than that of the previous year.

2. CANDIDATES' STRENGTHS

The Chief Examiner for Mathematics (Elective) listed the strengths of the candidates as ability to:

(i) rationalize a given surd;
(ii) find the mean and standard deviation from a given data;
(iii) find the resultant and magnitude of given vectors;
(iv) resolve given forces into component form;
(v) find the determinant and inverse of a given matrix.

3. **CANDIDATES' WEAKNESSES**

The Chief Examiner for Mathematics (Elective) listed some of the weaknesses of candidates as difficulty in:

(i) finding the coordinates of a point using internal division of a line in a given ratio;
(ii) solving problems on mechanics;
(iii) solving quadratic and linear equations simultaneously;
(iv) using trigonometric identities to solve trigonometry question.

4. SUGGESTED REMEDIES

The Chief Examiner for Mathematics (Elective) suggested that teachers should:

(i) give equal attention to the whole syllabus rather than specializing in few areas in the syllabus;
(ii) expose candidates to a lot of questions as exercises for them to have a good command on the topics in the syllabus;
(iii) encourage students to show all steps used in solving a question clearly without jumping steps by using calculator;
(iv) coach students on the importance of accuracy so that they would work with at least four decimal places and round off answers after the whole calculation;
(v) Stress on the need to read and understand the demand of the question before answering.

5. DETAILED COMMENTS

Question 1

Two independent events *K* and *L* are such that p(*K*) = *x*, *P*(*L*) = (x + $\frac{1}{5}$) and *P*(*K*∩*L*) = $\frac{3}{20}$. Find the value of *x*.

Most candidates could not apply the multiplication law to solve the problem. They rather added the two expressions and therefore got the value of *x* wrong.

Candidates were required to solve the question as follows:
Since events *K* and *L* are independent
$P(K).\ P(L) = P(K \cap L)$
$x(x + \frac{1}{5}) = \frac{3}{20}$
$x^2 + \frac{1}{5}x = \frac{3}{20}$
$20x^2 + 4x - 3 = 0$
$(2x + 1)(10x - 3) = 0$
$x = -\frac{1}{2}$ or $\frac{3}{10}$
$\therefore x = \frac{3}{10}$

Question 2

Seven participants in an art contest were ranked by two judges as follows:

Participants	A	B	C	D	E	F	G
1^{st} Judge	3	4	1	6	5	7	2
2^{nd} Judge	3	6	2	5	7	4	1

(a) **Calculate, correct to three decimal places, the Spearman's rank correlation coefficient for scores of the judges.**

(b) **Comment on your results**.

Candidates were to calculate spearman's rank correlation coefficient and comment on the results; most candidates did well by finding the d^2 values and answered the question well. However, few candidates quoted the formula wrongly.

Candidates were required to solve the question as follows:
(a)

R_1	3	4	1	6	5	7	2
R_2	3	6	2	5	7	4	1
d_i	0	-2	-1	1	-2	3	1
d_i^2	0	4	1	1	4	9	1

$P = 1 - \frac{6\sum d^2}{n(n^2-1)}$
$= \quad 1 - \frac{6(20)}{7(7^2-1)}$
$= \quad 0.6429$
= 0.643 (correct to three decimal places).

(b) There exists a positive correlation between the ranks of the two judges.

<u>Question 3</u>

F_1 (3N, 030º), F_2 (4N,090º), F_3 (6N,135º) and F_4 (7 N, 240º) act on a particle. Find, correct to two decimal places, the magnitude of the resultant force.

Most candidates did well by resolving the forces and continued to find the resultant force and magnitude. Only few candidates failed to leave the answer in two decimal places as required by the question.

Candidates were required to solve the question as follows:

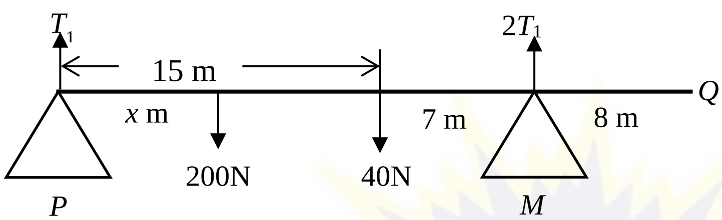
Let R be the resultant force.
$R = \begin{pmatrix} 3\sin 30° \\ 3\cos 30° \end{pmatrix} + \begin{pmatrix} 4\sin 90° \\ 4\cos 90° \end{pmatrix} + \begin{pmatrix} 6\sin 135° \\ 6\cos 135° \end{pmatrix} + \begin{pmatrix} 7\sin 240° \\ 7\cos 240 \end{pmatrix}$
$R = \begin{pmatrix} 1.5 \\ 2.5981 \end{pmatrix} + \begin{pmatrix} 4 \\ 0 \end{pmatrix} + \begin{pmatrix} 4.24426 \\ -4.2426 \end{pmatrix} + \begin{pmatrix} -6.0621 \\ -3.5 \end{pmatrix}$
$R = \begin{pmatrix} 3.6804 \\ -5.1445 \end{pmatrix}$
$|R| = \sqrt{(3.6804)^2 + (-5.1445)^2}$
= 6.3254 N
= 6.33N (correct to **two** decimal places).

Question 4

A uniform pole, *PQ*, 30m long and of mass 4 kg is carried by a boy at *P* and a man 8 m away from *Q*. Find the distance from *P* where a mass of 20kg should be attached so that the man's support is twice that of the boy, if the system is in equilibrium. [Take g = 10ms^{-2}]

This question was poorly done by candidates who attempted it. The use of principles of moments was the focus of the question. The few candidates who attempted it were not able to sketch the diagram, hence the distance from *P* was wrongly found.

Candidates were required to solve the question as follows:



Given that $T_2 = 2T_1$, so that

$T_1 + 2T_1 = 200 + 40$

$3T_1 = 240$

$T_1 = 80$ N

Taking moment about *P*,

$40 \times 15 + 200\,x = 2T_1 \times 22$

$600 + 200x = 2 \times 80 \times 22$

$200x = 2920$

$x = 14.6$m.

Question 5

Solve: $3x^{½} + 5 – 2x^{–½} = 0$

Most candidates who attempted this question had difficulty in removing the exponents. The question was poorly answered.

Candidates were required to solve the question as follows:

Let $m = x^{½}$

$\Rightarrow 3m + 5 – {}^{2}/m = 0$

$3m^2 + 5m – 2 = 0$

$(3m – 1)(m + 2) = 0$

$m = \frac{1}{3}$ or $m = –2$

$\therefore x = (\frac{1}{3})^2 = \frac{1}{9}$ or $x = (–2)^2 = 4$

Question 6

A point P divides the straight line joining $X\,(1, -2)$ and $Y\,(5, 3)$ internally in a ratio 2:3. Find the:

(a) coordinates of P.

(b) equation of the straight line that passes through $N\,(3, -5)$ and P.

(a) Most candidates attempted this question. Scores were very low since most of them could not find the coordinates of P.

Candidates were required to solve the question as follows:

Let $P\,(x, y)$ be the coordinates of P.

$P\,(x, y) = \left(\frac{3(1)+2(5)}{3+2}, \frac{2(3)+3(-2)}{3+2}\right)$

$= \left(\frac{3+10}{5}, 0\right)$

$= \left(\frac{13}{5}, 0\right)$

(b). The equation of the line was poorly done.

Candidates were required to solve the question as follows:

Gradient = $\frac{0-(-5)}{\frac{13}{5}-3}$

$= -\frac{25}{2}$

Equation of the line is given as

$y - 0 = -\frac{25}{2} - (x - \frac{13}{5})$

$2y = -5(5x-13)$

$2y + 25x - 65 = 0$

Question 7

(a) Find the sum of the series: 32 + 8 + 2 +….

(b) Simplify: $\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}} - \frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}$.

(a) Candidates exhibited good and clear understanding of the concept of finding the common ratio. Candidates were required to solve the question as follows:

$S_\infty = \frac{32}{1-\frac{1}{4}}$

$= 32\times\frac{4}{3}$

$= 42\frac{2}{3}$

(b) Most candidates rationalized the expression correctly and were able to simplify the surds. Candidates were required to solve the question as follows:

$\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}} - \frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} = \frac{(1-\sqrt{2})(\sqrt{5}-\sqrt{3})-(1+\sqrt{2})(\sqrt{5}-\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})}$

$= \frac{2\sqrt{3}-2\sqrt{10}}{2}$

$= \sqrt{3} - \sqrt{10}$

Question 8

Without using Mathematical tables or calculator, find, in surd form (radicals), the value of tan 22.5º.

Scores of the candidates were extremely poor on this question. Few candidates who attempted it did not seem to have a good understanding of the concept of double or compound angle. Most of the candidates could not establish the needed quadratic equation.

Candidates were required to solve the question as follows:

$\frac{2\tan A}{1-\tan^2 A}$

Let $2A = 45°$, therefore $A = 22.5°$

$\tan 45° = \frac{2\tan 22.5°}{1-\tan^2 22.5°}$

$1-\tan^2 22.5° = 2\tan 22.5°$

Let $\tan 22.5° = x$

$1 - x^2 = 2x$

$x^2 + 2x - 1 = 0$

$x = \frac{-2\pm\sqrt{4-4(-1)}}{2}$

$= 1 \pm \sqrt{2}$

$\therefore \tan 22.5° = 1 + \sqrt{2}$

Question 9

(a) Find the range of values of x for which $2x^2 \geq 9x + 5$.

(b) (i) Write down in ascending powers of x the binomial expansion of $(2+x)^6 - (2-x)^6$.

(ii) Using the result in (b) (i), evaluate $(2.01)^6 - (1.99)^6$, correct to four decimal places.

Most candidates attempted this question. In part (a), candidates factorized the given quadratic correctly. However most of them were unable to investigate for the conditions under which it was valid.

Candidates were required to solve the question as follows:

(a) $2x^2 \geq 9x + 5$

$2x^2 - 9x - 5 \geq 0$

$(2x + 1)(x - 5) \geq 0$

$x \leq -\frac{1}{2}$ or $x \geq 5$

(b) This part was not well done by most of the candidates; they did not show clear understanding of the concept and use of the binomial theorem.

Candidates were required to solve the question as follows:

(i) $(2 + x)^6 = 64 + 192x + 240x^2 + 160x^3 + 60x^4 + 12x^5 + x^6$

$(2 - x)^6 = 64 - 192x + 240x^2 - 160x^3 + 60x^4 - 12x^5 + x^6$

$\therefore\ (2 + x)^6 - (2 - x)^6 = 384x + 320x^3 + 24x^5$.

(ii) $2 + x = 2.01$ so that $x = 0.01$

Similarly, $2 - x = 1.99$ so that $x = 0.01$

$(2.01)^6 - (1.99)^6 = 384(0.01) + 320(0.01)^3 + 24(0.01)^5$

$= 3.84 + 0.00032 + 0.0000000024$

$= 3.8403$ (4 d.p.)

<u>Question 10</u>

A circle $x^2 + y^2 - 2x - 4y - 5 = 0$ with centre O is cut by a line $y = 2x + 5$ at points P and Q. Show that $\overline{QO}$ is perpendicular to $\overline{PO}$.

Most of the candidates did not attempt this question. It was very unpopular. They were to find centre of the circle, coordinates of point of intersection of the line and the circle and show perpendicular between two lines. Few candidates attempted this question.

Candidates were required to solve the question as follows:

$x^2 + y^2 - 2x - 4y - 5 = 0$

Substitute $2x + 5 = y$ into the equation of the circle

$x^2 + (2x + 5)^2 - 2x - 4(2x + 5) - 5 = 0$

$x^2 + 4x^2 + 20x + 25 - 2x - 8x - 20 - 5 = 0$

$5x^2 + 10x = 0$

$x^2 + 2x = 0$

$x(x + 2) = 0$

$x = 0$ or $x = -2$

When $x = 0$, $y = 2(0) + 5 = 5$

When $x = -2$, $y = 2(-2) + 5 = 1$

Therefore, P (x, y) = (0, 5) and Q (x, y) = (–2, 1)
Center of circle $x^2 + y^2 - 2x - 4y - 5 = 0$ is (1, 2)
Gradient of $\overline{QO} = \frac{2-1}{1-(-2)}$
$= \frac{1}{3}$
Gradient of $\overline{PO} = \frac{5-2}{0-1}$
$= -3$
Product of gradients = $\frac{1}{3} \times (-3) = -1$
Therefore, $\overline{QO}$ is perpendicular to $\overline{PO}$.

Question 11

(a) Given that M = $\begin{pmatrix} 3 & -5 \\ 4 & 2 \end{pmatrix}$, find:

(i) M^{-1}, inverse of M.

(ii) the image of (1, - 1) under M^{-1}.

(b) Two linear transformations *P* and Q are defined by *P*: $(x, y) \rightarrow (5x + 3y, 6x + 4y)$ and *Q*:$(x, y) \rightarrow (4x - 3y, -6x + 5y)$.

(i) Write down the matrices *P* and *Q*.

(ii) Find the matrix *R* defined by *R* = *PQ*.

(iii) Deduce Q^{-1}, the inverse of *Q*.

(a) Most candidates who answered this question performed well. The determinant and inverse were well found.

Candidates were required to solve the question as follows:

(i) M = $\begin{pmatrix} 3 & -5 \\ 4 & 2 \end{pmatrix}$
Determinant of $M = 6 + 20 = 26$
Then, $M^{-1} = \frac{1}{26}\begin{pmatrix} 2 & 5 \\ -4 & 3 \end{pmatrix}$
(ii) image = $\frac{1}{26}\begin{pmatrix} 2 & 5 \\ -4 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$
$= \frac{1}{26}\begin{pmatrix} -3 \\ -7 \end{pmatrix}$
Therefore, image of (1, –1) under $M^{-1} = \left(\frac{-3}{26}, -\frac{7}{26}\right)$

(b) Only few candidates could not write matrices *P* and *Q* correctly. The product *PQ* was well demonstrated.

Candidates were required to solve the question as follows:
(i) $P = \begin{pmatrix} 5 & 3 \\ 6 & 4 \end{pmatrix}$ and $Q = \begin{pmatrix} 4 & -3 \\ -6 & 5 \end{pmatrix}$

(ii) $R = \begin{pmatrix} 5 & 3 \\ 6 & 4 \end{pmatrix}\begin{pmatrix} 4 & -3 \\ -6 & 5 \end{pmatrix}$

$= \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

(iii) Let $QQ^{-1} = I$

$PQ = R$

$PQ = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

$PQ = 2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\Rightarrow \frac{1}{2}PQ = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\Rightarrow Q^{-1} = \frac{1}{2}P$

$Q^{-1} = \frac{1}{2}\begin{pmatrix} 5 & 3 \\ 6 & 4 \end{pmatrix}$

<u>Question 12</u>

A box contains 5 blue, 7 green and 4 red identical balls. Three balls are picked from the box one after the other without replacement. Find, the probability of picking:

(a) two green balls and a blue ball;

(b) no blue ball;

(c) at least one green ball;

(d) three balls of the same colour.

This question was popular with candidates, but scores were below average. In part (a),(b) and (c) most of the candidates could not use the binomial probability distribution to arrive at the required solutions. Knowledge and use of the binomial probability distribution seem to be a difficult concept for candidates. However, most of them were able to do the part (d) very well.

Candidates were required to solve the question as follows:

(a) P (**two** green and **a** blue ball) = $\frac{\binom{7}{2}\times\binom{5}{1}}{\binom{16}{3}} = \frac{21\times5}{560}$

$= \frac{3}{16}$ or 0.1875

(b) P (**no** blue ball) = $\frac{\binom{11}{3}}{\binom{16}{3}} = \frac{165}{560} = \frac{33}{112}$ or 0.2946

(c) P (**at least one** green ball) = 1 – P (no green ball)

$= 1 - \frac{\binom{9}{3}}{\binom{16}{3}}$

$= 1 - \frac{84}{560}$

$= \frac{17}{20}$ or 0.85

(d) P (**three** balls of the same colour) = $\frac{\binom{5}{3}+\binom{7}{3}+\binom{4}{3}}{\binom{16}{3}}$

$= \frac{10+35+4}{560}$

$= \frac{7}{80}$ or 0.0875

<u>Question 13</u>

The ages *x*, (in years), of a group of 18 adults have the following statistics:

$\boldsymbol{\Sigma x = 745}$ **and** $\boldsymbol{\Sigma x^2}$ **= 33951**

(a) Calculate the:

- **(i) mean age;**
- **(ii) standard deviation of the ages of the adults, correct to two decimal places.**

(b) One person leaves the group and the mean age of the remaining 17 is 41 years. Find the:

- **(i) age of the person who left;**
- **(ii) standard deviation of the remaining 17 adults, correct to two decimal places.**

Candidates who attempted this question scored low marks. Most candidates correctly found the mean and the standard deviation, but few candidates quoted the formula wrongly. The part (b) was poorly done because most of the candidates could not find the age of the person who left, hence standard deviation was poorly calculated.

Candidates were required to solve the question as follows:

(a) (i) Mean age = $\frac{745}{18}$ = 41.3889

(ii) Standard deviation = $\sqrt{\frac{33951}{18} - \left(\frac{745}{18}\right)^2}$

$= \sqrt{173.1265}$

= 13.16 (2 d.p.)

(b) (i) Let *x* be the age of the person who left the group.

$\frac{745-x}{17} = 41$

$x = 745 - 697$

$x = 48$ years

(ii) The sum of squares for the remaining 17 adults in the group is

$\Sigma x^2 = 33951 - (48)^2 = 31647$

Standard deviation = $\sqrt{\frac{31647}{17} - 48^2}$

$= \sqrt{180.5882}$
$= 13.44$(2 d.p.)

Question 14

Three forces 0i – 63j, 32.14i + 38.3j and 14i – 24.25j act on a body of mass 5 kg. Find, correct to the nearest whole number, the:

(a) magnitude of the resultant force;

(b) direction of the resultant force;

(c) acceleration of the body.

Almost every candidate attempted this question. Candidates could find the magnitude and direction of resultant force correctly. On the part (c), the acceleration was well calculated. The performance was above average.

Candidates were required to solve the question as follows:

(a) Let F_R be the resultant force.

$F_R = \begin{pmatrix} 0 \\ -63 \end{pmatrix} + \begin{pmatrix} 32.14 \\ 38.3 \end{pmatrix} + \begin{pmatrix} 14 \\ -24.25 \end{pmatrix}$

$F_R = \begin{pmatrix} 46.14 \\ -48.95 \end{pmatrix}$

$|F_R| = \sqrt{(46.14)^2 + (48.95)^2}$

$= \sqrt{4524.997}$

= 67.2681 N

= 67 N (**nearest** whole number)

(b) $\tan\theta = \frac{48.95}{46.14} = 1.0609$

$\theta = \tan^{-1}(1.0609)$

$= 46.69°$

The direction of the resultant force = 90° + 46.69° = 136.69°

= 137° (**nearest** whole number)

(c) Acceleration of the body $= \frac{67.2681}{5}$

= 13ms^{-2} (**nearest** whole number)

Question 15

Two particles *P* and *Q* move toward each other along a straight-line *MN*, 51 metres long. *P* starts from *M* with velocity 5ms^{-1} and constant acceleration of 1 ms^{-2}. *Q* starts from *N* at the same time with velocity 6 ms^{-1} and at a constant acceleration of 3 ms^{-2}. Find the time when the:

(a) particles are 30 metres apart;

(b) particles meet;

(c) velocity of $\boldsymbol{P}$ is $\frac{3}{4}$ of the velocity of $\boldsymbol{Q}$.

This question was not very popular with candidates. Few candidates attempted this question. Candidates were required to solve the question as follows:

(a) Let S_p be the distance of particle P and S_Q be the distance of the particle Q.

$S_p = 5t + \frac{1}{2}t^2$

$S_Q = 6t + \frac{3}{2}t^2$

$6t + \frac{3}{2}t^2 + 5t + \frac{1}{2}t^2 = 21$

$2t^2 + 11t - 21 = 0$

$(2t - 3)(t + 7) = 0$

$t = 1.5$ s

(b) $6t + \frac{3}{2}t^2 + 5t + \frac{1}{2}t^2 = 51$

$2t^2 + 11t - 51 = 0$

$(2t + 17)(t - 3) = 0$

$t = 3$ s

(c) $V_Q = 6 + 3t$

$V_P = 5 + t$

$5 + t = \frac{3}{4}(6 + 3t)$

$20 + 4t = 18 + 9t$

$t = \frac{2}{5}$ second or 0.4 seconds.